

IFT-96/14

hep-ph/9608321

# CONSTRAINING 2HDM BY PRESENT AND FUTURE $(g - 2)_\mu$ DATA

MARIA KRAWCZYK

*Institute of Theoretical Physics, University of Warsaw, ul.Hoża 69  
Warsaw, 00-681, Poland*

JAN ŻOCHOWSKI

*Institute of Theoretical Physics, University of Warsaw, ul.Hoża 69  
Warsaw, 00-681, Poland*

February 1, 2008

## Abstract

Constraints on the general 2HDM ("Model II") are obtained from the existing  $(g - 2)_\mu$  data including limits on Higgs bosons masses from LEP I data. We consider separately two cases: with a light scalar  $h$  and with a light pseudoscalar  $A$ , assuming  $M_h + M_A \geq M_Z$ . The charged Higgs contribution is also included. It is found that already the present  $(g - 2)_\mu$  data improve limits obtained recently by ALEPH collaboration on  $\tan\beta$  for the mass of the pseudoscalar below  $\lesssim 2$  GeV. The improvement in the accuracy by factor 20 in the forthcoming E821 experiment may lead to more stringent, than

provided by ALEPH group, limits up to  $M_A \sim 30$  GeV if the mass difference between  $h$  and  $A$  is  $\sim M_Z$ . Similar results should hold for a light scalar scenario as well.

# 1 Status of 2HDM.

## 1.1 Introduction.

The mechanism of spontaneous symmetry breaking proposed as the source of mass for the gauge and fermion fields in the Standard Model (SM) leads to a neutral scalar particle, the minimal Higgs boson. According to the LEP I data, based on the Bjorken process  $e^+e^- \rightarrow HZ^*$ , it should be heavier than 66 GeV[1], also the MSSM neutral Higgs particles have been constrained by LEP1 data to be heavier than  $\sim 45$  GeV [2, 3, 1]. The general two Higgs doublet model (2HDM) may yet accommodate a very light ( $\lesssim 45$  GeV) neutral scalar  $h$  or a pseudoscalar  $A$  as long as  $M_h + M_A \gtrsim M_Z$  [2].

The minimal extension of the Standard Model is to include a second Higgs doublet to the symmetry breaking mechanism. In two Higgs doublet models the observed Higgs sector is enlarged to five scalars: two neutral Higgs scalars (with masses  $M_H$  and  $M_h$  for heavier and lighter particle, respectively), one neutral pseudoscalar ( $M_A$ ), and a pair of charged Higgses ( $M_{H^+}$  and  $M_{H^-}$ ). The neutral Higgs scalar couplings to quarks, charged leptons and gauge bosons are modified with respect to analogous couplings in SM by factors that depend on additional parameters :  $\tan\beta$ , which is the ratio of the vacuum expectation values of the Higgs doublets  $v_2/v_1$ , and the mixing angle in the neutral Higgs sector  $\alpha$ . Further, new couplings appear, e.g.  $Zh(H)A$  and  $ZH^+H^-$ .

In this paper we will focus on the appealing version of the models with two doublets ("Model II") where one Higgs doublet with vacuum expectation value  $v_2$  couples only to the "up" components of fermion doublets while the other one couples to the "down" components [5]. In particular, fermions couple to the pseudoscalar  $A$  with a strength proportional to  $(\tan\beta)^{\pm 1}$  whereas the coupling of the fermions to the scalar  $h$  goes as  $\pm(\sin\alpha/\cos\beta)^{\pm 1}$ , where the sign  $\pm$  corresponds to isospin  $\mp 1/2$  components. In such model FCNC processes are absent and the  $\rho$  parameter retains its SM value at the tree level. Note that in such scenario the large ratio  $v_2/v_1 \sim m_{top}/m_b \gg 1$  is

naturally expected.

The well known supersymmetric model (MSSM) belongs to this class. In MSSM the relations among the parameters required by the supersymmetry appear, leaving only two parameters free (at the tree level) e.g.  $M_A$  and  $\tan\beta$ . In general case, which we call the general 2 Higgs Doublet Model (2HDM), masses and parameters  $\alpha$  and  $\beta$  are not constrained by the model. Therefore the same experimental data may lead to very distinct consequences depending on which version of two Higgs doublet extension of SM, supersymmetric or nonsupersymmetric, is considered.

## 1.2 Present constraints on 2HDM from LEP I.

Important constraints on the parameters of two Higgs doublet extensions of SM were obtained in the precision measurements at LEP I. The current mass limit on charged Higgs boson  $M_{H^\pm} = 44$  GeV/c was obtained at LEP I [4] from process  $Z \rightarrow H^+H^-$ , which is independent on the parameters  $\alpha$  and  $\beta$ . (Note that in the MSSM version one expects  $M_{H^\pm} > M_W$ ). For neutral Higgs particles  $h$  and  $A$  there are two main and complementary sources of information at LEP I. One is the Bjorken processes  $Z \rightarrow Z^*h$  which constrains  $g_{hZZ}^2 \sim \sin^2(\alpha - \beta)$ . The second process is  $Z \rightarrow hA$ , constraining the  $g_{ZhA}^2 \sim \cos^2(\alpha - \beta)$  for  $M_h + M_A \lesssim M_Z$ <sup>1</sup>. This Higgs pair production contribution depends also on the masses  $M_h$ ,  $M_A$  and  $M_Z$ .

Results on  $\sin^2(\alpha - \beta)$  and  $\cos^2(\alpha - \beta)$  can be translated into the limits on neutral Higgs bosons masses  $M_h$  and  $M_A$ . In the MSSM, due to relations among parameters, the above data allow to draw limits for the masses of individual particles:  $M_h \geq 45$  GeV for any  $\tan\beta$  and  $M_A \geq 45$  GeV for  $\tan\beta \geq 1$  [3, 1]. In the general 2HDM the implications are quite different, here the large portion of the  $(M_h, M_A)$  plane, where both masses are in the range between 0 and  $\sim 50$  GeV, is excluded [2].

The third basic at LEP I process in search of a neutral Higgs particle is the Yukawa process, *i.e.* the bremsstrahlung production of the neutral Higgs boson  $h(A)$  from the heavy fermion,  $e^+e^- \rightarrow f\bar{f}h(A)$ , where  $f$  means here  $b$  quark or  $\tau$  lepton. This process plays a very important role since it constrains the production of a very light pseudoscalar even if the pair

---

<sup>1</sup> The off shell production could also be included, *e.g.* as in [3].

production is forbidden kinematically <sup>2</sup>, *i.e.* for  $M_h + M_A > M_Z$ . It allows also to look for a light scalar, being an additional and in case of  $\alpha = \beta$  the most important, source of information. The importance of this process was stressed in many papers [6, 7], the recent discussion of the potential of the Yukawa process is presented in Ref.[8].

New analysis of the Yukawa process by ALEPH collaboration [9] led to the exclusion plot (95%) on the  $\tan\beta$  versus the pseudoscalar mass,  $M_A$ . (Analysis by L3 collaboration is also in progress [10].) It happened that obtained limits are rather weak <sup>3</sup>, allowing for the existence of a light  $A$  with mass below 10 GeV with  $\tan\beta = 20\text{--}30$ , for  $M_A=40$  GeV  $\tan\beta$  till 100 is allowed! For mass above 10 GeV, similar exclusion limits should in principle hold also for a scalar  $h$  with the replacement  $\tan\beta \rightarrow \sin\alpha/\cos\beta$ . Larger differences one would expect however in region of lower mass, where the production rate for the scalar is considerably larger than for the pseudoscalar and therefore more stringent limits should be obtained [8].

### 1.3 The 2HDM with a light Higgs particle.

In light of the above results from precision experiments at LEP I there is still a possibility of the existence of one light neutral Higgs particle with mass below  $\sim 40\text{--}50$  GeV. As far as other experimental data are concerned, especially these from low energy measurements, they do not contradict this possibility as they cover only part of the parameter space of 2HDM, moreover some of them like the Wilczek process have large theoretical uncertainties both due to the QCD and relativistic corrections [11, 5] (see also discussion in [12, 13, 14]).

In following we will study the 2HDM assuming that one light Higgs particle may exist. Moreover we will assume according to LEP I data the following mass relation between the lightest neutral Higgs particles:  $M_h + M_A \geq M_Z$ . We specify the model further by choosing particular values for the parameters  $\alpha$  and  $\beta$  within the present limits from LEP I. Since  $\sin(\alpha - \beta)^2$  was found [2, 1] to be smaller than 0.1 for the  $0 \lesssim M_h \lesssim 50$  GeV, and even below 0.01 for a lighter scalar, we simply take  $\alpha = \beta$ . This assumption leads to equal in strengths of the coupling of fermions to scalars and pseudoscalars.

---

<sup>2</sup>neglecting the off shell production

<sup>3</sup>Note, that the obtained limits are much weaker than the limits estimated in Ref. [8].

For the scenario with large  $\tan\beta \sim \mathcal{O}(m_t/m_b)$  large enhancement in the coupling of both  $h$  and  $A$  bosons to the down-type quarks and charged leptons is expected.

As we described above the existing limits from LEP I for a light neutral Higgs scalar/pseudoscalar boson in 2HDM are rather weak. Therefore it is extremely important to check if more stringent limits can be obtained from other measurements. The possible effect due to such a light Higgs particle at  $ep$  collider HERA and at low energy LC, as well in heavy ion collisions at HERA and LHC are discussed elsewhere [12, 15, 16, 17].

In this paper we study in details the limits on the 2HDM from the precision  $(g - 2)$  experiment for muon<sup>4</sup>. We have studied this problem earlier using the simple approach constraining the individual contributions  $h$  (or  $A$ ) (see Ref.[13, 14, 15] and also [7]). In the present analysis we take into account the full contribution from 2HDM, i.e. exchanges of  $h$ ,  $A$  and  $H^\pm$  bosons incorporating the present constraints on Higgs bosons masses from LEP I. We study here the present data on  $(g - 2)_\mu$  [18] as well as the potential of the future E821 experiment [19] with the accuracy expected to be more than 20 times better than the present one.

## 2 Constraints on the parameters of 2HDM from $(g - 2)$ for the muon.

### 2.1 The room for new physics.

The present experimental data limits on  $(g - 2)$  for muon averaged over the sign of the muon electric charge is given by [21]:

$$a_\mu^{exp} \equiv \frac{(g - 2)_\mu}{2} = 1\,165\,923\,(8.4) \cdot 10^{-9}.$$

The quantity within parenthesis,  $\sigma_{exp}$ , refers to the uncertainty in the last digit. The expected high-precision E821 Brookhaven experiment has design sensitivity of  $\sigma_{exp}^{new} = 4 \cdot 10^{-10}$  (later even  $1-2 \cdot 10^{-10}$ [33]) instead of the above  $84 \cdot 10^{-10}$ . It is of great importance to reach this accuracy in the theoretical analysis.

---

<sup>4</sup> Measurement of  $(g - 2)$  of the electron does not give useful restriction due to small mass of the electron (see also [35]).

The theoretical prediction of the Standard Model for this quantity consist of the QED, hadronic and EW contribution:

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{had} + a_\mu^{EW}.$$

The recent SM calculations of  $a_\mu$  are based on the QED results from [22, 23], hadronic contribution obtained in [25, 24, 26, 28, 30, 29, 33] and [31] and the EW results from [33, 32]. The uncertainties of these contributions differ among themselves considerably (see *e.g.* discussion below and in Ref.[20, 33, 27]). The main discrepancy is observed for the hadronic contribution, therefore we will consider here two cases : case A <sup>5</sup> with relatively small error in the hadronic part and the case B <sup>6</sup> with the 2 times larger error in the hadronic part. (We adopt here the notation from [20] but one should be aware that the numbers used in this analysis differ slightly from Ref.[20]. Basically our case A corresponds to case A from [20], whereas case B based on the [33].)

<i>case</i>	A [in 10 <sup>-9</sup> ]	B [in 10 <sup>-9</sup> ]
QED	1 165 847.06 (0.02)	1 165 847.06 (0.02)
had	69.70 (0.76)	68.82 (1.54)
EW	1.51 (0.04)	1.51 (0.04)
tot	1 165 918.27 (0.76)	1 165 917.39 (1.54)

Note that the hadronic contribution error dominates the total error for the SM predictions and will influence strongly the comparison with the new precision data from E821, what will be discussed later.

The room for a new physics is given basically by the difference between the experimental data and theoretical SM prediction:  $a_\mu^{exp} - a_\mu^{SM} \equiv \delta a_\mu$ . In the following we will assume that it is consistent to use this difference  $\delta a_\mu$  as the indication for the contribution due to Higgs particle(s) of a beyond SM origin, like 2HDM, although in the calculation of  $a_\mu^{EW}$  the (SM) Higgs scalar contribution is included. This assumption based on the observation that two EW calculations: the EW result from Ref.[33] based on  $M_{Higgs}=250$  GeV and

---

<sup>5</sup>Refs.[22, 23, 25, 24, 30, 33]

<sup>6</sup>Refs.[22, 23, 26, 31, 33]

the one from Ref.[32] with  $M_{Higgs} \geq 5$  GeV, differ only by  $\sim 0.02(0.06) \cdot 10^{-9}$ .  
7

Below the difference  $\delta a_\mu(\sigma)$  for these two cases: A and B, is presented together with the error  $\sigma$ , obtained by adding in quadrature the experimental and theoretical errors: *i.e.*  $\sigma = \sqrt{(\sigma_{exp}^2 + \sigma_{tot}^2)} \sim \sqrt{(\sigma_{exp}^2 + \sigma_{had}^2)}$ .

case	A [in $10^{-9}$ ]	B [in $10^{-9}$ ]
$\delta a_\mu(\sigma)$	4.73(8.43)	5.61(8.54)
lim(95%)	$-11.79 \leq \delta a_\mu \leq 21.25$	$-11.13 \leq \delta a_\mu \leq 22.35$
lim $_{\pm}$ (95%)	$-13.46 \leq \delta a_\mu \leq 19.94$	$-13.71 \leq \delta a_\mu \leq 20.84$

One can see that at one sigma level the difference  $\delta a_\mu$  can be positive or negative. For that beyond SM scenarios in which both positive and negative  $\delta a_\mu$  may appear, the 95% confidence level (C.L.) bound can be calculated straightforward (above denoted by  $lim(95\%)$ ). For the model where the contribution of only one sign is physically accessible, the other sign being unphysical, (*i.e.* positive or negative  $\delta a_\mu$ ) the 95%C.L. limits should be calculated in different way [21]. These limits calculated separately for the positive and for the negative contributions are denoted above by  $lim_{\pm}(95\%)$ .

We found that this latter approach leads to the sizeable shift in the lower ( $l_-$ ) and upper ( $l_+$ ) limits with respect to the standard (95%) limits (by  $-1.3 \cdot 10^{-9}$  up to  $-2.6 \cdot 10^{-9}$ ). That means that the possible negative contribution becomes larger whereas the positive becomes smaller when  $lim_{\pm}$  method is used. The differences between theoretical predictions (case A and B) for fixed method of calculating confidence level seem to be smaller than this effect. All these effects may be important in future analysis.

## 2.2 2HDM contribution to $(g - 2)_\mu$ .

As we mentioned above the difference between experimental and theoretical value for the anomalous magnetic moment for muon we ascribe to the 2HDM contribution, so in order to constrain the parameter space of the model we take  $\delta a_\mu = a_\mu^{(2HDM)}$ .

---

<sup>7</sup>The contribution due to the Higgs scalar for the  $M_{Higgs}=5$  GeV was found to be of the level of  $1 \cdot 10^{-11}$  [32].

To  $a_\mu^{(2HDM)}$  a scalar  $h$  ( $a_\mu^h$ ), pseudoscalar  $A$  ( $a_\mu^A$ ) and the charged Higgs boson  $H^\pm$  ( $a_\mu^\pm$ ) contribute. The relevant formulae can be found in the Appendix. Each term  $a_\mu^\Lambda$  ( $\Lambda = h, A$  or  $H^\pm$ ) disappears in the limit of large mass, at small mass the contribution reaches its maximum (or minimum if negative) value. The scalar contribution  $a_\mu^h(M_h)$  is positive whereas the pseudoscalar boson  $a_\mu^A(M_A)$  gives negative contribution, also the charged Higgs boson contribution is negative. Note that since the mass of  $H^\pm$  is above 44 GeV (LEP I limit), its small contribution can show up only if the sum of  $h$  and  $A$  contributions is small.

We calculate the  $a_\mu^{(2HDM)}$  minimalizing the cross section in order to put limit on the maximum allowed  $\tan\beta$ . Therefore we take  $M_\pm^0=44$  GeV <sup>8</sup> and for mass of neutral Higgs bosons we assume the relation  $M_h + M_A \geq M_Z$  as discussed before.

We study two scenarios:

- a) pseudoscalar  $A$  is light, the scalar mass  $M_h \geq M_Z - M_A$  and we calculate the total 2HDM contribution as follows:

$$a_\mu^{(2HDM)}(M_A) = a_\mu^A(M_A) + a_\mu^h(M_h^0 = M_Z - M_A) + a_\mu^\pm(M_\pm^0) \quad (1a)$$

- b) scalar  $h$  is light, the pseudoscalar mass is  $M_A \geq M_Z - M_h$  and:

$$a_\mu^{(2HDM)}(M_h) = a_\mu^h(M_h) + a_\mu^A(M_A^0 = M_Z - M_h) + a_\mu^\pm(M_\pm^0). \quad (1b)$$

Due to opposite signs there appear a cancellation in both scenarios between scalar and pseudoscalar contributions, especially strong for  $M_A \sim M_h^0$  in (1a) or  $M_h \sim M_A^0$  in (1b). Note that the total 2HDM contribution is for the scenario a) negative, whereas for the scenario b)– positive. Therefore we have to include this fact calculating the 95% C.L. bounds of  $a_\mu^{(2HDM)}$ . This leads us to the limits  $lim_\pm(95\%)$  described previously in Sec.2.1.

*Present data.*

---

<sup>8</sup>the case with mass equal 600 GeV was also studied, but in the range of masses of neutral Higgs bosons below 40 GeV discussed here, the influence of so heavy charged Higgs boson is negligible, see below.



Since the case A (Sec.2.1) gives more stringent  $lim_{\pm}(95\%)$  constraints, for both positive and negative contributions, only this case will be used in the further analysis. So, we have for the considered 2HDM scenarios allowed following ranges:

$$a) \quad -13.46 \cdot 10^{-9} \leq a_{\mu}^{(2HDM)} \leq 0 \quad (2a)$$

$$b) \quad 0 \leq a_{\mu}^{(2HDM)} \leq 19.94 \cdot 10^{-9}. \quad (2b)$$

Using Eqs.2a(2b) the 95% C.L. exclusion plots for  $\tan \beta$  versus  $M_A$  ( $M_h$ ) is calculated for a light pseudoscalar (light scalar) scenario. The results are presented in Fig.1 (2). The regions above curves are excluded.

The results based on the *current* data for  $(g-2)_{\mu}$  are presented by *upper* curves. The total contribution of the 2HDM (solid line) as well the simple approach where only pseudoscalar (scalar) contribution (short-dashed line) is included are presented in Fig.1 and 2. They lead to similar limits up to mass  $\sim 20$  GeV. Note that the simple approach for pseudoscalar (scalar) corresponds to the limits of negligible contribution of scalar (pseudoscalar), i.e. to the large difference in mass between  $h$  and  $A$  (above 150 GeV or so).

The charged Higgs boson contributes very little (in Figs.1 and 2 the difference between a solid line and the long-dashed line, calculated without the charged Higgs boson term) being visible only for masses of  $h$  and  $A$  above 40 GeV, where the cancellation between scalar and pseudoscalar becomes very strong (see small figures in Figs.1, 2 for details).

Interestingly, the present  $(g-2)_{\mu}$  data can accommodate large value of  $\tan \beta$  (20 or more) for the Higgs boson masses equal or larger than 2 GeV (see also the discussion in Ref.[7, 13, 15]).

*Forthcoming data.*

Since the dominate uncertainty in  $\delta a_{\mu}$  (Sec.2.1) is due to the experimental error, the role of the forthcoming E821 experiment is crucial in testing the SM or probing a new physics. We discuss now the potential of this measurement for the constraining the 2HDM.

The future accuracy of the  $(g-2)_{\mu}$  experiment is expected to be  $\sigma_{exp}^{new} \sim 0.4 \cdot 10^{-9}$  or better, and one may in principle calculate the expected error for

the difference  $\delta a_\mu^{new}$ . Assuming no progress in theoretical SM calculation the above experimental accuracy will lead to  $\sigma^{new} = 0.86$  and  $1.6$ , for case A and B, respectively.

However, especially for the case A discussed in Sec.2.1, one could in principle expect such an improvement in the calculation of the hadronic contribution<sup>9</sup> that the total uncertainty in  $\delta a_\mu$  will be basically due to the experimental error. We will take this point of view and in particular we will assume that the accessible range for  $a_\mu^{(2HDM)}$ , given presently by Eq.2a(2b) for a light pseudoscalar (scalar), would be smaller by factor 20. In this approach the central value for the difference  $\delta a_\mu$  is shifted by the same factor 20 to lower value  $0.24 \cdot 10^{-9}$  (from  $4.73 \cdot 10^{-9}$ ). So, we consider the following option (in  $10^{-9}$ ):

$$\delta a_\mu^{new} = 0.24, \quad \text{and} \quad \lim_{\pm}^{new}(95\%) : -0.69 \leq \delta a_\mu \leq 1.00$$

or separately for two scenarios:

$$a') \quad -0.69 \cdot 10^{-9} \leq a_\mu^{(2HDM)} \leq 0 \quad (3a)$$

$$b') \quad 0 \leq a_\mu^{(2HDM)} \leq 1.00 \cdot 10^{-9} \quad (3b)$$

As expected much stringent limits on  $\tan\beta$  are obtained in this case for pseudoscalar (scalar) – see Fig.1 (2) *lower* curves.

## 2.3 Discussion and comparison with ALEPH limits.

The comparison of the obtained 95%C.L. exclusion plots based on the current and future accuracy for the  $(g-2)_\mu$  data with the latest results from ALEPH collaboration based on the Yukawa process [9] is presented in Fig.3. In this figure a simple approach prediction for a light  $h$  (solid line) or a light  $A$  (long-dashed line) are shown for current and future  $(g-2)_\mu$  measurement, *upper* and *lower* curves, respectively.

---

<sup>9</sup>The improvement in the ongoing experiments at low energy is expected as well.

The results from ALEPH collaboration are presented by dotted line, only for the pseudoscalar case. At very low mass, below say 2 GeV, the present  $(g - 2)_\mu$  data leads to stronger limits on  $\tan \beta$  than the Yukawa process.

Similar results from Yukawa process as presented by dotted line should also hold for the case of a light scalar for  $M_h$  above 10 GeV. For lower mass both Yukawa and  $(g - 2)_\mu$  data may lead to comparable exclusions plots.

Note that the predictions based on the simple approach can be justified if the mass difference between  $h$  and  $A$  is bigger than, say 150 GeV. In this case the improved  $(g - 2)_\mu$  data for muon will lead to more stringent limits over the whole considered range of masses. If the mass difference between  $h$  and  $A$  is  $\sim M_Z$  the improvement may be obtained up to  $\sim 30$  GeV.

### 3 Conclusion

We studied the room for new physics in the current muon anomalous magnetic moment data based on the newest theoretical calculations of the SM prediction  $a_\mu^{(SM)}$ .

The difference between experimental and theoretical value for the anomalous magnetic moment for muon was ascribed to the 2HDM contribution, so we took  $\delta a_\mu = a_\mu^{(2HDM)}$ . From evaluated 95% C.L. bounds for the positive or negative value of  $a_\mu^{(2HDM)}$  we derived constraints on the general 2HDM ("Model II").

We studied the total 2HDM contribution  $a_\mu^{(2HDM)}$  separately in two cases: with a light scalar  $h$  ( $M_h \leq 45$ ) and with a light pseudoscalar  $A$  ( $M_h \leq 45$ ), assuming according to the LEP I data that  $M_h + M_A \geq M_Z$ . A light scalar scenario leads to the positive, whereas the one with a light pseudoscalar to the negative  $a_\mu^{(2HDM)}$ . Large cancellation occurs when both mass of neutral Higgs bosons are nearly degenerated  $M_h \sim M_A \sim M_Z/2$ . In this case the charged Higgs boson contribution may show up in the range of  $M_h(A)$  above 40 GeV, if we take the lower LEP I limit for  $M_\pm = 44$  GeV.

We found that the contribution due to a light  $A$  becomes larger whereas

this one due to a light  $h$  becomes smaller by  $\sim 1.5$  or even  $2.6 \cdot 10^{-9}$  if the proper evaluation of 95% C.L. is introduced. The differences between theoretical predictions (mainly due to the uncertainty in the hadronic corrections) on the level of  $0.25$  to  $0.9 \cdot 10^{-9}$  are smaller than this effect. Note also that the theoretical uncertainty influences less the lower limit, relevant for the pseudoscalar contribution, than the upper one, where a scalar contribution is expected. All these effects may be important for the future accuracy measurement of  $(g - 2)$  for muon.

It was found that already the present  $(g-2)_\mu$  data improve limits obtained recently by ALEPH collaboration on  $\tan\beta$  for low mass of the pseudoscalar:  $M_A \leq 2$  GeV. The future improvement in the accuracy by factor 20 in the forthcoming E821 experiment may lead to more stringent limits than provided by ALEPH group up to  $M_A = 30$  GeV or higher for a larger mass difference between scalar and pseudoscalar. Similar results should hold for a 2HDM with a light scalar.

## 4 Acknowledgements

We are grateful to Debayjoti Choudhury, Bohdan Grzadkowski, and Roman Nowak for useful discussions and comments. We thank Jan Kalinowski for pointing us Ref.[26]. Many thanks to J.-M. Grivaz, P. Janot and J.-B. de Vivie for sending us the ALEPH results concerning the Yukawa process. Supported in part by the Polish Committee for Scientific Research.

## 5 Appendix

Theoretical predictions of an anomalous magnetic moment of muon from triangle vertex with the exchange of the Higgs boson  $\Lambda$  ( $= h, A$  or  $H^\pm$ ) are given in Ref.[36, 35].

$$a_\mu^\Lambda = \frac{f_\Lambda^2}{8\pi^2} \tilde{L}_\Lambda,$$

where the coupling constant  $f_\Lambda$  is given by

$$f_\Lambda \equiv \frac{g}{2} \frac{m_\mu}{M_W} \tan \beta,$$

(in case of scalar  $\tan \beta$  should be replace by the  $\sin(\alpha)/\cos(\beta)$ , but in our model we put  $\alpha = \beta$ , so the coupling constant is universal ).

The integral  $\tilde{L}_{h(A)}$  is for the neutral Higgs contribution given by:

$$\tilde{L}_{h(A)} = \int_0^1 dx \frac{Q(x)}{x^2 + (1-x)(M_{h/A}/m_\mu)^2}.$$

with

$$Q_h(x) = x^2(2-x) \tag{1}$$

$$Q_A(x) = -x^3. \tag{2}$$

The charged Higgs particle exchange is described by:

$$\tilde{L}_\pm = \int_0^1 dx \frac{-x(1-x)}{x + (M_\pm/m_\mu)^2 - 1}.$$

## References

- [1] P. Maettig, Plenary Talk at ICHEP96, Warsaw, July 1996.
- [2] ALEPH Coll., D. Buskulic et al., *Z. Phys.* **C 62** (1994) 539;  
DELPHI Collab., P. Abreu et al., *Nucl. Phys.* **B 418** (1994) 403;  
L3 Collab., M. Acciarri et al., *Z. Phys.* **C 62** (1994) 551; Note 1800 (1995)  
OPAL Collab. R. Akers et al., *Z. Phys.* **C 61** (1994) 19.
- [3] ALEPH Coll. Note EPS0415(1995);  
L3 Coll. Note 1801(1995).

- [4] A. Sopczak, talk at *IX Workshop on High Energy Physics, Moscow 1994* (hep-ph/9504300).
- [5] J.F. Gunion et al., *The Higgs Hunter's Guide*, (Addison-Wesley,1990).
- [6] . J. Kalinowski and S. Pokorski, *Phys. Lett.* **B219** (1989) 116;  
J. Kalinowski, N. P. Nilles, *Phys. Lett.* **B 255** (1991) 134;  
A. Djouadi et al, *Phys. Lett.* **B259** (1990) 175.
- [7] E. Carlson, S. Glashow, U. Sarid, *Nucl.Phys.* **B309** (1988) 597
- [8] J. Kalinowski, M. Krawczyk, *Phys.Lett* **B361** (1995)66; *Acta Phys.Pol.* **B27** (1996) 961.
- [9] ALEPH Coll. *Search for a light Higgs Boson in the Yukawa Process*, contr. to ICHEP96, PA13-027.
- [10] A. Sopczak, private communication
- [11] Crystal Ball Collab., D. Antresyan et. al., *Phys. Lett* **B251** (1990) 204;  
CLEO Collab., R. Balest et al., CLEO 94-19;  
P. Franzini et al., *Phys. Rev.***35** (1987) 2883.
- [12] A.C. Bawa and M. Krawczyk, Warsaw University IFT preprints 16/91 (and ERRATUM) and 16/92; *Phys. Lett* **B357** (1995) 637; M. Krawczyk, contr. to Workshop "Future of HERA" 1995–1996.
- [13] M. Krawczyk, Warsaw University IFT preprint IFT-95/20, talk at Kazimierz Symposium (May 1995, Ames, USA).
- [14] M.Krawczyk, talk at Moriond'96 (EW session), IFT - 96/15. (hep-ph/9607268)
- [15] D. Choudhury, M. Krawczyk, MPI-Pth/96-46, IFT 95/21, in *Proc. Workshop NLC*, 1995, p.69; D. Choudhury, M. Krawczyk, MPI-PTh/96-46, IFT 96/13 (hep-ph/9607271)- submitted to Phys.Lett. B.

- [16] M. Krawczyk and B.B. Levchenko, *Two-photon mechanism production of the Higgs boson, SUSY particles, hadrons and lepton pairs in  $eA$  collision at HERA*, contr. to Workshop “Future of HERA” 1995–1996. M. Krawczyk and J. Żochowski, *Two-photon coherent production of the light Higgs boson at LHC*– in preparation.
- [17] M. Krawczyk, talk at ICHEP’96, (Warsaw, July 1996).
- [18] J. Bailey et al., *Phys. Lett.* **68** B (1977) 191;  
F. J. M. Farley and E. Picasso, *Ann. Rev. Nucl. Sci.* **29** (1979) 243;  
in *Quantum Electrodynamics*, ed. T. Kinoshita, (World Scientific, Singapore);  
F. J. M. Farley, *Z. Phys.* **C56** (1992) S88
- [19] *Proposal for Brookhaven National Laboratory Experiment E821: A precision measurement of the muon  $g - 2$  value at the level of 0.35 ppm*, E. Benedict et al. (1986), co-spokesmen V. W. Hughes, W. M. Morse; B. L. Roberts; B. L. Roberts, *Z. Phys.* **C56** (1992) S101 and talk at ICHEP’96, July 1996, Warsaw, Poland.
- [20] U. Chattopadhyay and P. Nath, *Phys. Rev.* **D53** (1996) 1648.
- [21] Particle Data Group, *Phys. Rev.* **D50** (1994) 1173.
- [22] T. Kinoshita et al., *Phys. Rev.* **D41** (1990) 593; T. Kinoshita, *Phys. Rev.* **D47** (1993) 5013; M. Samuel and G. Li, *Phys. Rev.* **D44** (1991) 3935 (Erratum **D48** (1993) 1879; S. Laporta and E. Remiddi, *Phys. Lett.* **B301** (1992) 440; S. Laporta, *Phys. Lett.* **B312** (1993) 495; S. Karshenboim, *Yad. Fiz.* **56** (1993) 252; S. Laporta and E. Remiddi, preprint DFUB-95-05 (hep-ph/9602417).
- [23] T. Kinoshita, *Phys. Rev. Lett.* **75** (1995) 4728.
- [24] T. Kinoshita and W. J. Marciano, in *Quantum Electrodynamics*, ed. by T. Kinoshita (World Scientific, Singapore, 1990), pp 419-478; T.

- Kinoshita et al, *Phys. Rev. Lett.* **31** (1985) 2108; T. Kinoshita, *Z. Phys.* **C56** (1992) S80.
- [25] L. Martinovic and S. Dubnicka, *Phys. Rev.* **D42** (1990) 884;
  - [26] S. Eidelman and F. Jegerlehner, *Z. Phys.* **C67** (1995) 585.
  - [27] F. Jegerlehner, in *Proc. of the Workshop “QCD and QED in Higher Orders”, Rheinsberg, Germany, 1996* (hep-ph/9606484).
  - [28] D. H. Brown and W. A. Worstell, *The lowest order hadronic contribution to the muon  $g - 2$  value with systematic error correlations*(hep-ph/9607319)
  - [29] E. de Rafael, *Phys. Lett.* **B322** (1994) 239.
  - [30] J. Bijmens et al., *Phys. Rev. Lett.* **75** (1995) 1447;
  - [31] M. Hayakawa et al., *Phys. Rev. Lett.* **75** (1995) 790; DPNU-95-30,(hep-ph/9601310).
  - [32] T. V. Kukhto et al., *Nucl. Phys.* **B371** (1992) 567.
  - [33] A. Czarnecki et al., *Phys. Rev. Lett.* **76** (1996) 3267; *Phys. Rev.* **D52** (1995) R2619.
  - [34] W. A. Bardeen et al. *Nucl. Phys.* **B46** (1972) 315.
  - [35] H. E. Haber et al., *Nucl. Phys.* **B161** (1979) 493.
  - [36] J. Leveille, *Nucl. Phys.* **B137** (1978) 63.



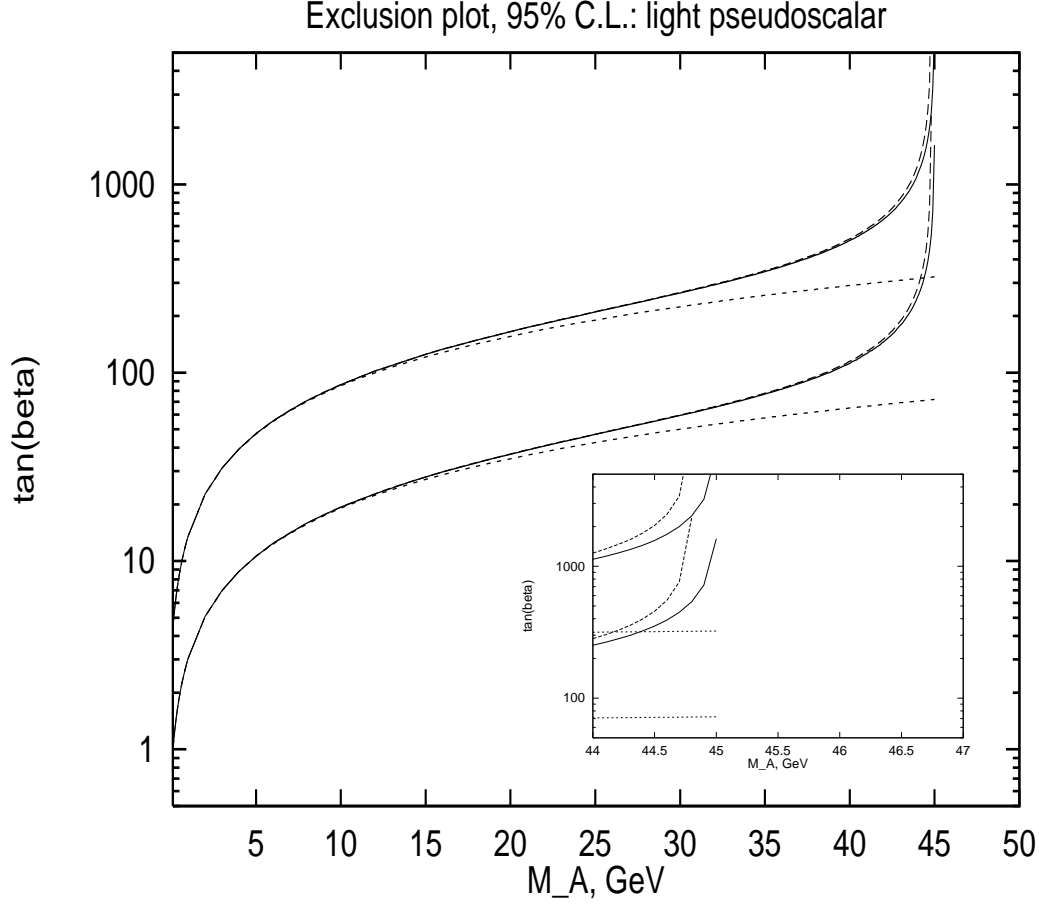


Figure 1: The 95% exclusion plot, based on  $\lim_{\pm}$ , from the  $(g - 2)_{\mu}$  data for pseudoscalar. The present limits (Eq.2a) (upper curves) and for the improved measurement (Eq.3a) (lower curves) are presented. The total 2HDM (Eq.1a) contribution is represented by the solid lines, the short-dashed lines correspond to the simple approach where only pseudoscalar contributes. The long-dashed lines correspond to the case where the charged Higgs particle contribution is neglected; for details see small figure, where the mass range above 44 GeV is displayed.

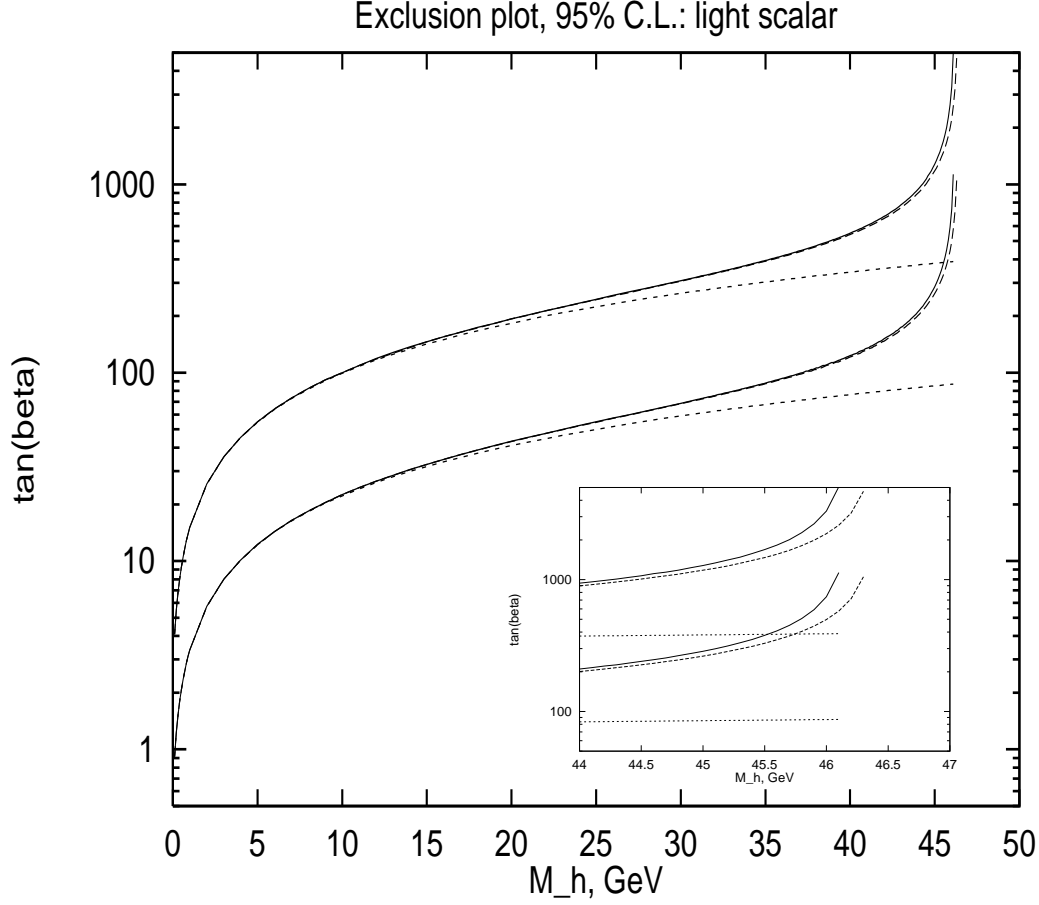


Figure 2: The 95% exclusion plot, based on  $\lim_{\pm}$ , from the  $(g - 2)_{\mu}$  data for scalar. The present limits (Eq.2b) (upper curves) and the improved measurement (Eq.3b) (lower curves) are presented. The total 2HDM (Eq.1b) contribution is represented by the solid lines, the short-dashed lines correspond to the simple approach where only scalar contributes. The long-dashed lines correspond to the case where the charged Higgs particle contribution is neglected; for details see small figure, where the mass range above 44 GeV is displayed.

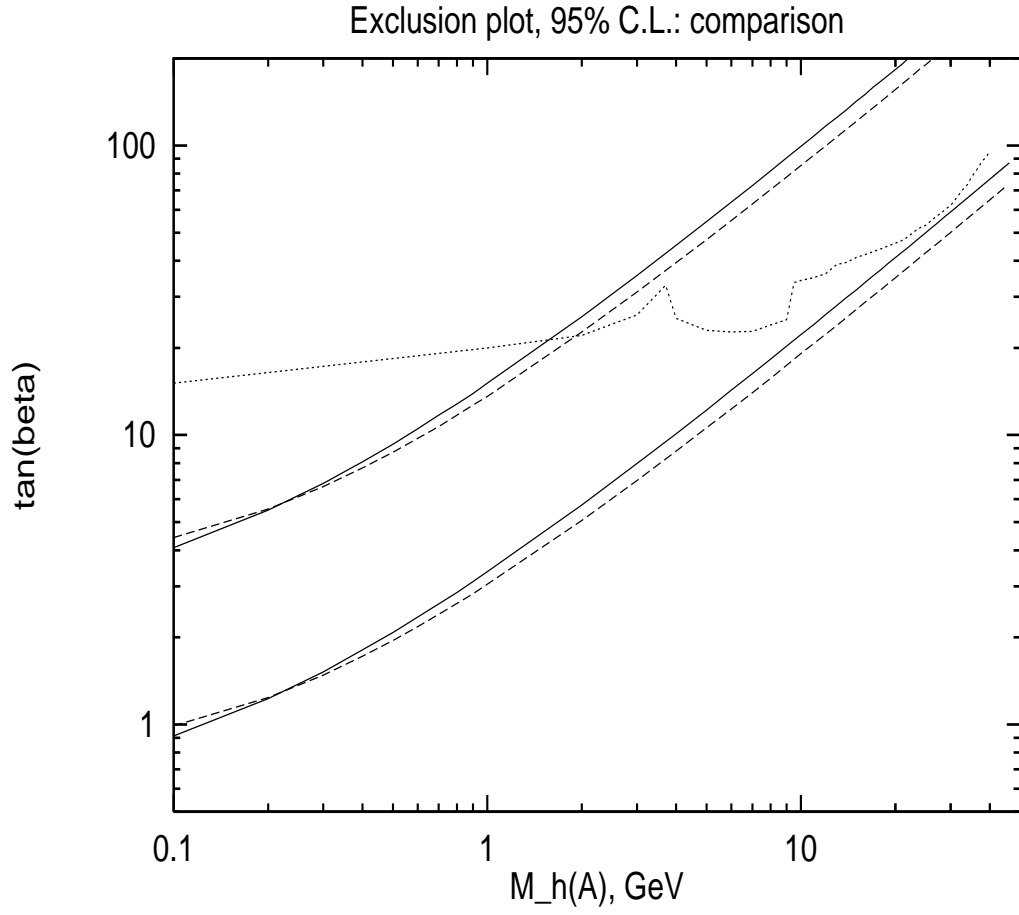


Figure 3: The 95% exclusion plot, based on  $\lim_{\pm}$ , from the present (upper curves) and future (lower curves)  $(g-2)_\mu$  data. The simple approach results for a light  $h$  (solid line), and a light  $A$  (long-dashed line) with the new ALEPH analysis for pseudoscalar (dotted line) are presented.